**Quadratics (2)**

**1. (a)** Determine the restrictions for such that the equation has real roots.

 **(b)** Determine the restrictions for such that the equation has real roots.

 **(a)**  has real roots if and only if

 **(b)**  has real roots if and only if

 (Check: If , the given equation is reduced to , it is no longer quadratic but has real root .)

**2.** **(a)** If and are the roots of the equation , where are real numbers and ,

 show that and .

 **(b)** If the above equation has real roots and are real numbers such that , show that

 the equation also has real solutions.

**(a)** Since and are the roots of the equation ,

 therefore

 or

 Compare coefficients of ,

 and .

**(b)** If the equation has real roots

 Therefore .

 For the equation ,

 Case 1, If , then , (given )

 ()

 Case 2, If , from (3), and multiply by ,

 In both cases, , therefore the equation also has real solutions.

**3.**  If and are the roots for the equation , where are real numbers,

 show that ,

 If and are the roots of , show that

 **(a)**

 **(b)**

 **(c)** Hence or otherwise, determine the relationship needed for the above two quadratic equations to have one **common** real root.

 ,

**(a)** By Vieta’s Theorem,

**(b)**

 Similarly in **(a)**, we have

**(c)** The above two equations to have one common real root

 Note that should be rejected since .

 In order that “The above two equations to have one common **real** root.”, we need :

 1. For ,

 2. For ,

 In conclusion,

 The above two equations to have one common real root

**4.** If and , prove that

 Hence, if α and β are the roots of the equation , show that

**Method 1**

 Since α and β are the roots of the equation

 and

 The equation with roots is

 or

**Method 2**

 Since

 By Vieta Theorem,

 Hence

**5.** If and are the roots of the equation .

 Prove that the equation with roots and is

 **Method 1**

 Hence,

 Therefore the equation with roots and is

 **Method 2**

Use the transformation or and put it in the given quadratic equation

 The required equation is therefore

**6.** If and are the roots for the equation and and are the roots for the equation where are real numbers, show that the equations with roots and is and show that the roots of this new equation are real.

 By Vieta Theorem, ,

 and

 New sum of roots =

 New product of roots =

 =

 =

 =

 =

 The new equation is therefore:

 Therefore the roots of this new equation are real.

 (If the new equation has equal roots)

**7.** If α and β are the roots of the equation form the equation where the roots are .

 **Method 1**

 Since α and β are the roots of the equation ,

 and

 New sum of roots =

 New product of roots =

 Therefore the equation where the roots are is

 or

 **Method 2**

 Since α and β are the roots of the equation ,

 and

 Similarly

 Therefore we use the transformation : .

 Therefore the equation where the roots are is

 (where y is the variable)

**8.** If one of the roots for the quadratic equation , is the positive square root of the other, show that

 Hence, find the value(s) of y such that the roots of the quadratic equation

 has a root that is the positive square root of the other.

 Let be the roots of the quadratic equation , .

 By Vieta Theorem,

 Since has a root that is the square root of the other, from

 Check : **(1)** If , becomes , then

 **(2)** If , becomes , then

 Since in both cases, one root is not the **positive** square root of the other, there is **no solution** for y .

**9.** If α is a root of the equation , where , form a quadratic equation with roots

 .

 Hence the quadratic equation with roots is

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